THE THERMODYNAMIC EXPANSION OF A HEATED GAS

V. F. Prisnyakov

Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 4, pp. 487–490, 1966

UDC 536.7

The thermodynamic expansion index is determined for a gas being evacuated from a reservoir with allowance for heat transfer.

A technical problem frequently encountered is the determination of the parameters of a gas issuing from a reservoir. The published papers dealing with this matter [1–3, 5–7] examine isolated particular cases, and only in [7] does the reader find some mean index of polytropic expansion.

In the present paper an expression is derived for the thermodynamic index during evacuation of a gas reservoir, with allowance for heat transfer due to free convection, as pointed out in [7].

The quantitative relations between the individual thermodynamic parameters of a free system are determined by the coefficient $\alpha = \Delta u/q$, which indicates the fraction of heat expended in increasing the internal energy of the gas. Ordinarily, we use not the coefficient α, but the related coefficient of polytropy, the exponent in the thermodynamic process equation. It should be noted that the coefficient of polytropy is presumed to be constant during the whole process.

In actual processes α is a variable quantity. Therefore the index of the process will vary with time, i.e., it is not an ideal thermostatic process that occurs, but a thermodynamic process in the full sense of the word.

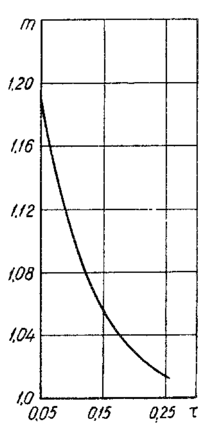

Fig. 1. Dependence of the thermodynamic index m of the process of expansion of a gas on time τ (sec) during evacuation of a reservoir.

In our case, that of evacuation of a gas reservoir with heat transfer, the expansion process may be represented by the geometrical location of points situated between an isotherm and an adiabat, each point corresponding to a definite time from the beginning of evacuation of the reservoir. In order to establish the nature of the process at different times, and

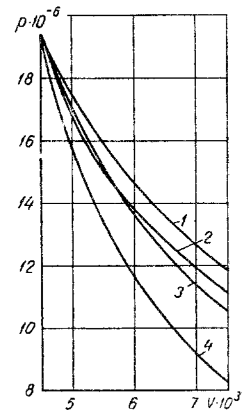

Fig. 2. The thermodynamic process of expansion of a gas at various values of m: 1) isothermal process (m = 1); 2) process of expansion of a gas, during evacuation of a reservior, when $m = m(\tau)$; 3) polytropic process, corresponding to the mean coefficient of polytropy, calculated from the formula of reference [7]; 4) adiabatic process, $m = k$ (p in N/m^2, V in m^3/kg).

also to make a comparative analysis of different processes, it is therefore expedient to use the concept of an instantaneous thermodynamic index, determined by the relation [5]

$$m = \frac{dp}{p} \Big/ \frac{d\rho}{\rho}. \tag{1}$$

This index is the "instantaneous characteristic of the law of the process, and should be regarded as the most important element in thermodynamic analysis" [5]. Therefore, using the results of [2], we shall determine its value for the case of evacuation of a reservoir, making allowance for heat transfer between the gas and the walls.

With the aid of the equation of state, we transform (1) to the form

$$m = 1 + \frac{\rho}{T}\frac{dT}{d\rho}. \tag{2}$$

Hence it may be seen that, to determine m, it is necessary to find the derivative $dT/d\rho$ and the relations

$p = p(\tau)$ and $\rho = \rho(\tau)$. For this it is necessary to integrate the equation describing variation of the gas parameters during evacuation of the reservoir [5],

$$\frac{dp}{d\tau} = \frac{k-1}{V}\left(\frac{dQ}{d\tau} - iG\right),$$

$$\frac{dp}{d\rho} = -\frac{(k-1)}{G}\left(\frac{dQ}{d\tau} - iG\right).$$

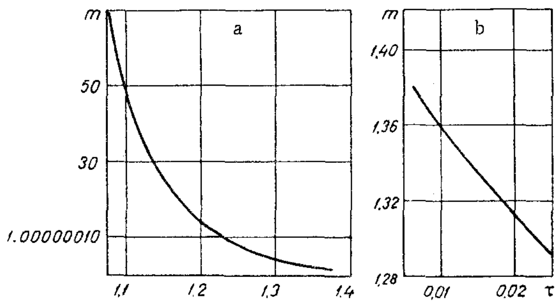

Fig. 3. Dependence of m on time τ (sec) of evacuation of the reservoir: a) with D_{cr} = = 0.004 m; b) 0.1 m.

Certain assumptions, described in [2], allow the integration to be performed, yielding the following time dependences of the pressure, density, and temperature of the gas issuing from the reservoir:

$$\bar{p} = \frac{p}{p_0} = \frac{a^2}{\vartheta_2^{\delta/2}}\frac{\mathrm{sh}^\delta(\omega\tau+\varepsilon)}{\mathrm{th}^2(\omega\tau+\varepsilon)}, \quad (3)$$

$$\bar{\rho} = \rho/\rho_0 = \vartheta_2^{-\delta/2}\mathrm{sh}^\delta(\omega\tau+\varepsilon), \quad (4)$$

$$\bar{T} = T/T_0 = a^2\,\mathrm{cth}^2(\omega\tau+\varepsilon), \quad (5)$$

in addition to the equation of the thermodynamic process

$$\frac{\bar{p}}{\bar{\rho}} - \frac{1}{1+\vartheta_2}\bar{\rho}^{-(k-1)(1+\vartheta_2)} - a^2 = 0. \quad (6)$$

Here

$$\omega = \frac{1}{2}a\vartheta_1(1+\vartheta_2)\sqrt{T_0},$$

$$\vartheta_1 = (k-1)A_k \cdot \mu F_{cr}\sqrt{R}/V;$$

$$\vartheta_2 = \frac{1{,}01\chi F}{A_k\mu F_{cr}\sqrt{R}}\sqrt{\frac{T_0}{p_0}},$$

$$a = \sqrt{\vartheta_2/(1+\vartheta_2)}, \quad \varepsilon = \mathrm{arth}\,a, \quad \delta = -\frac{2}{(k-1)(1+\vartheta_2)},$$

$$A_k = \sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}, \quad \chi = \chi_1\sqrt{T}.$$

The connection between T and ρ may easily be found, by eliminating time from (4) and (5):

$$\bar{T} = \frac{1}{(1+\vartheta_2)}\left[\vartheta_2 + \bar{\rho}^{-(k-1)(1+\vartheta_2)}\right]. \quad (7)$$

Hence we determine

$$\frac{d\bar{T}}{d\bar{\rho}} = (k-1)\bar{\rho}^{-(k-1)(1+\vartheta_2)-1} \quad (8)$$

Since

$$\frac{\rho}{T}\frac{dT}{d\rho} = \frac{\bar{\rho}}{\bar{T}}\frac{d\bar{T}}{d\bar{\rho}},$$

substituting (8) into (2) and taking (4) and (5) into account, we obtain

$$m = 1 + \frac{(k-1)(1+\vartheta_2)}{1+\vartheta_2\bar{\rho}^{-(k-1)(1+\vartheta_2)}}. \quad (9)$$

This expression may easily be transformed with the help of (4) into a dependence of the index of the thermodynamic process under examination upon time:

$$m = 1 + (k-1)\frac{(1+\vartheta_2)}{\mathrm{ch}^2(\omega\tau+\varepsilon)}. \quad (10)$$

From this special cases may be obtained. In an adiabatic process, where there is no heat transferred, $\chi_1 = 0$, $\vartheta_2 = 0$, $a = 0$, $\varepsilon = 0$, $\mathrm{ch}(\omega\tau+\varepsilon) = 1$, which gives m = k. In an isothermal process $\chi_1 \to \infty$, $\vartheta_2 \to \infty$, $a \to 1$, $\omega \to \infty$. Removing the indeterminary obtained in (10) according to l'Hôpital's rule, we find that m = 1.

Figure 1 shows a graph of variation of the index m with time during evacuation of a reservoir with T_0 = = 300° K, $p_0 = 196.1 \cdot 10^5$ N/m², F = 0.941 m², R = = 295 J/kg · degree, D_{cr} = 0.03 m, $V = 7.07 \cdot 10^{-2}$ m³, $\chi = 5.7$ kg$^{1/3}$/cek$^{5/3}$ · m$^{2/3}$ degree$^{5/6}$.

Figure 2 depicts an actual gas expansion process (curve 2) on the p-V diagram. Also shown for comparison are isothermal and adiabatic processes (curves 1 and 4), as well as a process (curve 3) corresponding to a mean coefficient of polytropy ($\bar{n}$ = 1.09) calculated from the formula of [7]. Depending on the time of evacuation of the reservoir, the gas expansion process will approximate either to the isothermal or the adiabatic. Figure 3 shows the influence of reservoir evacuation time on the nature of the process. For a small cross section, corresponding to a long evacuation time, the process is practically isothermal (a), while for a large cross section it tends to adiabatic.

NOTATION

p—pressure; ρ—density; m—index of thermodynamic process; T—temperature; τ—time; k—adiabatic exponent; V—volume of reservoir; i—enthalpy; G—mass flow rate of gas per second; Q—heat supplied; α—thermodynamic coefficient; u—internal energy; q—amount of heat supplied to 1 kg of gas; μ—flow rate coefficient; F_{cr}—area of section determining gas flow rate; F—internal surface area of reservoir; R—gas constant; χ_1—experimental coefficient obtained at a pressure of 1 bar and proportional to temperature to the power −1/2 (see [4]). The subscript 0 denotes parameters at time zero.

REFERENCES

1. B. N. Bezhanov, Pneumatic Mechanisms [in Russian], Mashgiz, 1957.
2. N. P. Belik, N. M. Belyaev, and G. S. Shandorov, IFZh, no. 9, 1964.
3. E. V. Gerts and G. V. Kreinin, Theory and Design of Pneumatic Power Equipment [in Russian], Izd. AN SSSR, 1960.
4. S. S. Kutateladze and V. M. Borishanskii, Heat Transfer Handbook [in Russian], GEI, 1959.
5. M. A. Mamontov, Topics in the Thermodynamics of Variable-Mass Bodies [in Russian], Oborongiz, 1961.
6. N. N. Markevich, Uch. zap. LGU, no. 27, s. matem. nauk, no. 31, 1957.
7. I. I. Morozov, IFZh, no. 12, 1959.
8. A. S. Yastrembskii, Technical Thermodynamics [in Russian], GEI, 1953.

17 September 1965

Tricentenary Ukraine-Russia Union State University, Dnepropetrovsk